7. J. Imberger, "Natural convection in a shallow cavity with differentially heated end walls. Part 3: Experimental results," J. Fluid Mech., 65, Pt. 2 (1974).
8. D. E. Cormack, G. P. Stone, and L. G. Leal, "The effect of upper surface conditions on convection in a shallow cavity with differentially heated end walls," Int. J. Heat Mass Transfer, 18, 635 (1975).
9. A. Bejan, A. A. Al-Homoud, and J. Imberger, "Experimental study of high-Rayleigh-number convection in a horizontal cavity with different end temperatures," J. Fluid Mech., 109, 283 (1981).
10. P. G. Simkins and T. D. Dudderar, "Convection in rectangular cavities with differentially heated end walls," J. Fluid Mech., 110, 433 (1981).
11. G. Shiralker and C. L. Tien, "Numerical study of laminar free convectiion in shallow closed cavities," Teploperedacha, No. 2 (1981).
12. G. Shiralker, A. Gadgil, and C. L. Tien, "High Rayleigh number convection in shallow enclosures with different end temperatures," Int. J. Heat Mass Transfer, 24, No. 10 (1981).
13. J. C. Launay, J. Miroglio, and B. Roux, "Contributions to the study of natural convection in a sealed tube for the growing of a single crystal by gas phase transport," J. Cryst. Growth, 51, 61 (1981).
14. L. D. Landau and E. M. Lifshits, Mechanics of Continuous Media [in Russian], GITTL, Moscow (1954).
15. A. V. Buné, V. L. Gryaznov, K. G. Dubovik, and V. I. Polezhaev, "Methods and computer programs for the numerical modeling of hydrodynamical processes by the nonstationary Navier—Stokes equations," Preprint No. 173, IPM, Academy of Sciences of the USSR (1981).
16. A. A. Samarskii, Theory of Differences [in Russian], Nauka, Moscow (1977).
17. V. I. Polezhaev and V. L. Gryaznov, "Method of calculating boundary conditions for the Navier—Stokes equations in terms of the vorticity and stream function variables," Dokl. Akad. Nauk SSSR, 219, No. 2 (1974).
18. R. A. Lodiz and R. L. Parker, Growth of Single Crystals [Russian translation], Mir, Moscow (1974).

LONGITUDINAL CELLULAR STRUCTURES OF TAYLOR—GÖRTLER TYPE VORTICES ON THE HIGH-PRESSURE SIDE OF ROTATING CHANNELS

L. V. Kuz'minskii, E. M. Smirnov, and S. V. Yurkin UDC 532.516

The linear stability of Poiseuille flow between two plates rotating about an axis parallel to the plates and normal to the direction of the basic flow was studied in [1, 2]. It was shown that the flow is least stable to disturbances in the form of standing waves, known as Taylor—Görtler vortices. The results of the experimental determination of the region of flow parameters for the formation of above vortices are given in [1] for a channel with rectangular section, strongly stretched along the axis of rotation. Experimental results agree well with linear theory. At the same time, it remains unclear about the important question of the effect of side walls on the stability of the basic flow in practically interesting cases of channels with moderate aspect ratio and, in particular, in channels with square cross section. The problem is formulated below and the results of experimental study of this problem are discussed.

The restructuring of the basic flow due to rotation was studied analytically and numerically in [3-6], assuming developed flow in rectangular cross-sectional channel. Here, as usual, developed flow means that the flow is sufficiently far from the entrance where the initial conditions are completely "forgotten" and the flow characteristics are identical at all cross sections.

As characteristic parameters for the channel with the given aspect ratio $\kappa = h/l$ we choose Reynolds number $Re = w_m l/\nu$ and the rotation parameter $K = \omega l/w_m$, where h and l are the

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 129-134, November-December, 1983. Original article submitted October 13, 1982.

 0021-8944/83/2406-0882$07.50

lengths of the cross section with the side of length h being parallel to the axis of rotation; w_m is the velocity based on mean mass flow.

The concrete form of the flow in the channel modified by rotation depends on the values and ratio of Re and K. However, it is possible to highlight the following general and the most characteristic features of the restructuring process. Secondary flow, whose development is caused by nonuniform distribution of Coriolis forces along the lines parallel to the axis of rotation, takes the form of a pair of vortices, the central plane of the channel perpendicular to the axis of rotation being the plane of symmetry. The relative transverse component of velocity attains a maximum when the quantities $Re_\omega = \omega h^2/\nu = \kappa^2 K Re$ have a value of the order 10 and when $Re_\omega >> 1$, it decreases approximately as $Re^{-1/2}_\omega$ [5]. At large κ the secondary flow is concentrated near the narrow sides. The effect of the secondary flow on the mean flow comes out in the form of the displacement of the maximum streamwise velocity component in the direction of increased pressure (flow toward the wall) and in the formation of the core in which the distribution of streamwise and one of the transverse velocity components are uniform along the directions parallel to the axis of rotation.

The formation of the core even in the case of channel with an aspect ratio of the order of one makes it possible to consider, in the qualitative analysis, a significant region as unbounded plane parallel flow with shear in a direction perpendicular to the axis of rotation. Damping or amplification of disturbances occur in such a flow, similar to the well-studied flows with circular streamlines. The flow is destabilized in the region in which the scalar product of the angular velocity and absolute vorticity vectors is negative and, on the other hand, it is more stable where this product is positive. In the destabilization region the mean flow is least stable to disturbances in the form of streamwise vortices, viz., Taylor–Görtler vortices. Computations [1, 2] show that for a given Re, disturbance of this type grow only in a limited range of the parameter K, which extends with an increase in Re. It is useful to mention that the wavelength of the most unstable disturbance decreases with an increase in K.

Results of many studies on related flow in the gap between two circular cylinders, with the inner one rotating, lead to the natural conclusion that even in the case of the flow along a rotating channel of aspect ratio of the order one, the effect of side walls which was neglected in the qualitative analysis of the type of instability, significantly alters the condition for the growth of disturbances when compared to the idealized problem [1, 2].

Four rectangular cross-sectional channels made of transparent Plexiglas were used for an experimental flow visualization study. The channels were sequentially set up on a frame rotating about a vertical axis with an angular velocity ω = 0-3 rad/sec. The channel axis was located in a plane normal to the axis of rotation. It is known that in isothermal incompressible flows parallel momentum transfer in the given flow field does not affect the characteristics of the velocity field in the rotating coordinate system [7]. For the sake of convenience of conducting the experiments, the channel axis was kept away from the axis of rotation by 259-300 mm. The necessary electronic instrumentation and the optical system were also set up on the rotating frame. The majority of the experiments were conducted using water bubbles [8] based on the electrochemical breakdown of water. Tap water from which dissolved gases were initially removed was used as the working medium. Water was circulated by a hydraulic system with automatic pumping into the reservoir. A special setup was used to transfer the fluid from the rotating frame. The mass flow was measured from the pressure drop in the measuring plate located in the stationary system and the error in the determination of mass flow was 3%.

The square cross-sectional channel was 30 × 30 × 800 mm. The end walls of the channel were extended to two identical rectangular chambers 140 × 100 × 40 mm, whose narrow sides were parallel to the channel axis. The water inlet (outlet) to the chamber was carried out through a 20-mm-diameter orifice drilled in the chamber wall perpendicular to the channel. An insert was installed at the channel entrance, establishing the initial conditions for the flow. Two types of inserts were used. The first type was of the type of a honeycomb made up of a set of thin-walled tubes (2-mm diameter and 15 mm long) and 1-3 screens with different mesh sizes. The second insert comprised of rectangular parallelepiped of length 30 mm made of flexible polyurethane. The flow visualization was carried out in a region 450 mm away from the insert exit.

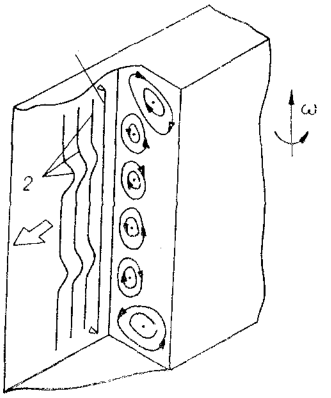

Fig. 1

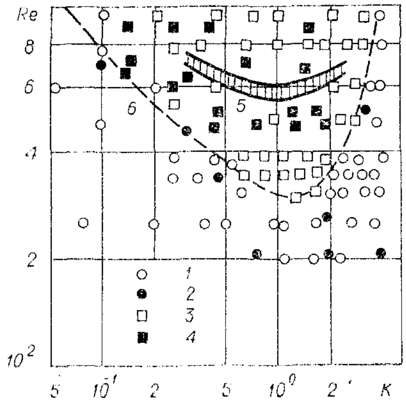

Fig. 2

Three other channels, each of length 800 mm, had rectangular cross sections h × l of 90 × 45 mm, 90 × 22.5 mm, and 90 × 12.5 mm; the aspect ratio of the sections was κ = 2, 4, and 7.2. These channels were not equipped with end chambers but were covered by soundproof lids. Water inlet (outlet) to the channel was carried out through orifices of 20-mm-diameter drilled into the wide wall of the channel with the center at a distance 30 mm from the end plates. At a distance 60 mm downstream from the end plates, a 40-mm-long polyurethane insert was installed. Flow visualization was carried out in a region at least 400 mm away from the insert exit.

In using the water bubble technique, a 20-μm tungsten wire, used as the cathode, was introduced into the channel through tubes with seals (Fig. 1, line 1). Anode, made of a stainless steel plate, was located downstream at a distance 60-90 mm from the test section. A special generator supplied 0.2-50 msec impulse at 25-100 V. The impulse frequency was 0.5-10 Hz. When the periodic impulse voltage is fed, a series of marker particles, viz., hydrogen bubbles, separate from thin tungsten wire and are entrained by the flow (Fig. 1, lines 2). These rows of hydrogen bubbles form local streamlines since the bubble diameter, not exceeding the wire diameter, is small and the relaxation as well as buoyancy effects of the bubbles are negligible.

A camera rotating along with the channel photographed the projections of local streamlines on the plane parallel to the side with higher pressure. The chosen range of angular velocity made it possible to conduct direct visualization of local streamlines.

A mixture of aniline ink with spirit was introduced as a dye with neutral buoyancy in the flow through the square cross-sectional channel. The dye came out of a small tube with an outer diameter of 0.7 mm inserted through the side with higher pressure into the central plane of the channel.

For the basic stable flow even a small rotation leads to linearity of streak lines coming from the wire parallel to the axis of rotation, except the short segment immediately close to the walls perpendicular to the wire. If the streak lines are generated by the wire located at a small distance from the leading wall, then the development of Taylor–Görtler vortices is accompanied by the formation of characteristic valleys in streak lines indicating the location of the flow directed from the wall into the channel (see Fig. 1). In certain cases the valleys are not aligned because of the increasing distance of streak lines from the wire and even becomes a peak of the same thickness. These changes in the configuration of streak lines are associated with the displacement of hydrogen bubbles along with the fluid in the region where there is a domination of longitudinal acceleration corresponding to the Coriolis component arising simultaneously with the development of vortices. The number of valleys (peaks) of streak lines indicate the number of pairs of Taylor–Görtler vortices.

The picture of streak lines formed as a result of the development of cellular vortex structure is stationary in a limited range of variation in Re and K. Starting from a certain value of Reynolds number, the lines connecting the center of valleys (peaks) go through an oscillating motion in the plane parallel to the leading wall and a small increase in Re results in a visually irregular flow structure. Similar changes also take place with the dye structure.

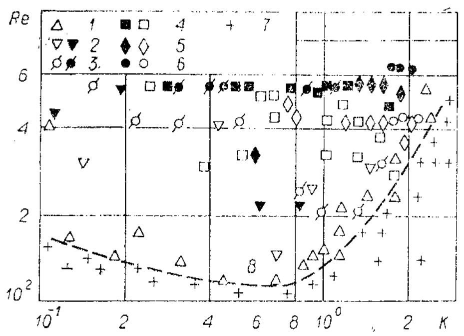

Fig. 3

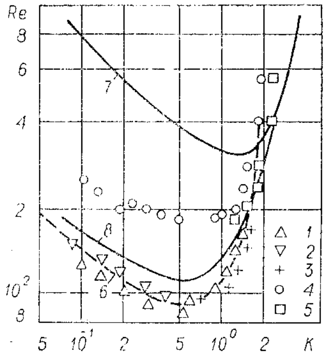

Fig. 4

Figure 2 shows results of flow visualization in the channel with square cross section. The numbers 1 and 2 indicate cases in which the formation of streamwise cellular structure of Taylor—Görtler vortices were not observed, 3 and 4 indicate the state of the flow with the development of disturbances along the high-pressure side without limitations on stationary and nonstationary conditions. The hatched region 5 represents the boundary above which vortices lose stationary character. The approximate curve 6 divides stable and unstable regions of the flow. The points 2 and 4 correspond to data obtained from hydrogen bubbles; 1 and 3 are the results from observations in dye. We also note that polyurethane inlet insert was used in flow visualization with hydrogen bubbles, whereas in the case of dye honeycomb structure was used as the inlet insert. It is seen from Fig. 2 that results from the two methods agree well.

In the experiments conducted in square cross-sectional channel, streak lines indicated the formation of only one pair of Taylor—Görtler vortices. Thus, a pair of vortex in the mean flow is supplemented by one more pair of streamwise vortices adjoining the high-pressure side. We note that the transition from two-vortex to four-vortex structure is also observed in the related case of curvilinear channel with square cross section [9]. The observations on dyes which describe a spiral motion showed that the area occupied by the supplementary vortices never exceeded half the channel section. The picture along the channel through the transparent walls of the end chamber were especially clear.

Figure 3 shows experimental data for channel with aspect ratio $\kappa = 4$. The numbers 1-6, respectively, indicate the number of pairs of Taylor—Görtler vortices formed. Filled circles indicate nonstationarity of streak line pattern. Points 7 correspond to flow conditions in which valleys in streak lines are not observed. The approximate curve 8 divides the stable and unstable regions of the basic flow. We note that during the passage of the characteristic parameters through critical values, realized in the experiments, as a rule by a gradual variation in angular velocity, the time for the growth or disappearance of valleys was quite large (up to 30 sec), which necessitated prolonged direct observations.

The tendency of the number of pairs of vortices to increase with K with Re = const, as well as with increase in Re with K = const, are considered in Fig. 3. The former tendency is in agreement with the results of linear analysis [1, 2] and the latter is caused by the weakening of end effects with increase in $Re_\omega = \kappa^2 K Re$.

The nonmonotonic change in the number of pairs of vortices formed is associated, apparently, with nonuniqueness of the possible stable conditions whose formation process is sensitive to the manner in which the chosen values of similar effects appearing as flow bifurcation, are excellently demonstrated in the study [10] of circular Couette flow in circular gap of small span. In connection with this, we observe that, in our experiments, the acceleration process, i.e., the establishment of the quantities Re and K, was not controlled to the necessary extent. The bifurcation phenomenon can also explain the nonstationarity of the streak line pattern for certain pairs of Re and K in the region below Re = 500, in which, for a majority of other conditions, the flow is stationary. Actually, in approaching the bifurcation curves in the Re—K plane, even a small disturbance can sharply change the flow to another state, but since a small oscillation in mass flow through the channel (up to 3%) was observed in the experiment, it, apparently, was the cause of the periodic establishment of one or the other type of flow, observed as nonstationarity of flow as a whole.

A more complete study of bifurcations in flows of this type requires the setting up of special and very subtle experiments.

Results of flow visualization in channels with $\kappa = 7.2$ (points 1-3) and $\kappa = 2$ (points 4 and 5) are given in Fig. 4. Studies were confined basically by visually fixing the beginning of the formation (curves 1 and 4) of one valley in the streak lines with a slow and gradual increase in angular velocity and later by observing conditions for the disappearance of the valley (curves 3 and 5) during the transition through the second branch of the stability boundary. The points 2 indicate the initial formation of two pairs of vortices in the channel with $\kappa = 7.2$.

Curve 6, plotted according to computed results [2], represents the geometric location of neutral stability points for the most dangerous disturbances in the rotating plane-parallel channel. It is seen that the experimental data for $\kappa = 7.2$ agree very well with the results of linear theory. Curves 7 and 8 are reproduced from Figs. 2 and 3 where they are indicated, respectively, by numbers 6 and 8. The observed increase in the stability limit for the basic flow with a decrease in κ is mainly due to a shift (caused by the secondary flow) in the maximum of the streamwise velocity component toward the high-pressure side, i.e., due to a decrease in the characteristic transverse dimension of the flow region where rotation has a destabilizing effect.

LITERATURE CITED

1. J. E. Hart, "Instability and secondary motion in a rotating channel flow," J. Fluid Mech., 45, 341 (1971).
2. D. K. Lezius, "Finite-difference solution of Taylor instabilities in viscous plane flow," Comput. Fluids, 3, 103 (1975).
3. H. Ludwieg, "Die ausgebildete Kanalströmung in einem rotierenden System," Ing. Archiv., 19, 296 (1951).
4. S. B. Nikol'skaya, "Laminar flow in rotating channels," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1977).
5. O. N. Ovchinnikov and E. M. Smirnov, "Flow dynamics and heat transfer in rotating channel," Inzh.-Fiz. Zh., 35, No. 1 (1978).
6. R. Schilling and H. Marcinowski, "Untersuchung des Druckverlustes und des Wärmeüberganges in rotierenden Kanälen mit rechteckigem Querschnitt," in: Recent Developments in Theor. and Experim. Fluid Mech., Berlin (1979).
7. Kh. Grinspen, Theory of Rotating Fluids [in Russian], Gidrometeoizdat, Leningrad (1975).
8. Shraub, Kline, et al., "Application of hydrogen bubbles in the quantitative study of time-varying velocity fields in low-speed water flows," Proc. ASME, Theor. Basic Eng., Design, 87, No. 2 (1965).
9. Chen, Lin, and Ou, "Fully developed laminar flow in curvilinear rectangular cross-sectional channels," Proc. ASME, Theor. Basic Eng. Designs, 98, No. 1 (1976).
10. T. B. Benjamin, "Bifurcation phenomena in steady flows of a viscous fluid. Pt. II. Experiment," Proc. R. Soc. London, A359, 27 (1978).